Stochastic Modeling: Exercise Class 6.

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Exercise 1. Describe the communication diagram, discuss the irreducibility, periodicity of the states, atoms and their accessibility, and invariant distribution(s) of the discrete Markov chains with transition matrix

- $K = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix}$,
- $K = \begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix}$,
- $K = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 0 & 0 & 1 \end{pmatrix}$.

Compute also $\mathbb{P}_1(\tau_1 = k)$ for $k = 1, 2, 3$.

Exercise 2. Consider the AR(1) model $X_{t+1} = \theta X_t + Z_{t+1}$ where $Z_1, Z_2, \ldots$ is an iid sample from the density $\mathcal{N}(0, 1)$ from $X_0 = x \in \mathbb{R}$ and $|\theta| < 1$.

- Show that the sequence (X_t) constitutes a Markov chain.
- Describe its kernel.
- Show that (X_t) is Lebesgue-irreducible.
- Show that (X_t) is atomless.
- Show that (X_t) satisfies the minorization condition.
- Describe its stationary distribution.