

# Time Series Analysis: TD1.

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## Exercise 1.

1. Give an example of second order stationary time series such that  $\gamma_X(h) = \sigma^2 > 0$  for any  $h \geq 0$ .
2. Give an example of strictly stationary times series which is not second order stationary.

**Exercise 2.** Consider  $(X, Y) \sim \mathcal{N}_2(0, \Sigma)$  with

$$\Sigma = \begin{pmatrix} \sigma^2 & c \\ c & \sigma^2 \end{pmatrix}$$

1. Check that  $\Sigma$  is a symmetric Toeplitz matrix. Under which condition on  $c$  it is positive? Assume it in the sequel.
2. Provide the covariance and the correlation  $\text{Cov}(X, Y)$  and  $\rho(X, Y)$ .
3. Compute the conditional density  $f_{X|Y=y}(x)$ .
4. Deduce that the best prediction of  $X$  given  $Y$  is  $\mathbb{E}[X | Y] = c/\sigma Y$ . What is the associate quadratic risk  $\mathbb{E}[(X - \mathbb{E}[X | Y])^2]$ ?
5. What is happening if  $c = 1$ ?

**Exercise 3.** Consider a gaussian WN(1)  $(Z_t)$  and the time series  $(X_t)$  defined as

$$X_t = \begin{cases} Z_t, & \text{if } t \text{ is even,} \\ (Z_{t-1}^2 - 1)/\sqrt{2}, & \text{if } t \text{ is odd.} \end{cases}$$

We recall that  $\mathbb{E}[Z_0^4] = 3$ .

1. Check that  $\mathbb{E}[Z_0^3] = 0$ .
2. Show that  $\mathbb{E}[X_t] = 0$  and  $\text{Var}(X_t) = 1$  for any  $t \in \mathbb{Z}$ .
3. Find the best predictions  $\mathbb{E}[X_n | X_{n-1}, \dots, X_1]$  for  $n$  odd and  $n$  even.
4. Show that  $(X_t)$  is WN(1) but not SWN(1).
5. Deduce that the best linear prediction  $\Pi_n(X_{n+1})$  equals 0 for any  $n \geq 1$ .
6. Compare the risk of the best predictions with the risk of the best linear prediction for  $n$  odd and  $n$  even.