Time Series Analysis: TD1.

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Exercise 1. Find examples of white noises which are not i.i.d., not stationary respectively.

Exercise 2.

- Cauchy-Schwarz: for any $f, g \in \mathbb{L}^2(\mathbb{P})$, $| < f, g > | \leq \|f\| \|g\|$,
- Show the triangular inequality $\|f + g\| \leq \|f\| + \|g\|$.

Exercise 3. Show that a MA(q) time series (X_t) is stationary as soon as (Z_t) is.

Exercise 4.

- Show that if $|\phi| > 1$, the equation $X_t = \phi X_{t-1} + Z_t$ admits a unique second order stationary solution of the form $X_t = -\sum_{j=1}^{+\infty} \phi^{-j} Z_{t+j}$. It is a linear time series that is not causal.
- Show that if $\phi = -1$, there is no second order stationary solution.

Exercise 5. We recall the Wold's theorem

Theorem 1 (Wold)**.** *Let (X_t) be second order stationary. Then X_t is uniquely decompose as*

$$X_t = \sum_{j \geq 0} \psi_j Z_{t-j} + r_t$$

where

- *(Z_t) is a WN(σ^2),*
- *$\psi_0 = 1$ and $\sum_{j \geq 0} \psi_j^2 < \infty$,*
- *$Cov(Z_t, r_s) = 0$ for all t, $s \in \mathbb{Z}$.*
- *(r_t) is deterministic.*

The aim of this exercise is to prove the two last points in order to complete the proof from the lecture notes

1. Define $r_t = X_t - \sum_{j \geq 0} \psi_j Z_{t-j}$ and show that $\mathrm{Cov}(Z_t, r_s) = 0$ for all t, $s \in \mathbb{Z}$.
2. Show that r_t belongs to the linear span of $X_{t-1}, X_{t-2}, \ldots$. Conclude.